QP CODE: 25050458

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Reg No : .....

Name : .....

MSc DEGREE (CSS)EXAMINATION DECEMBER 2025

First Semester

M Sc MATHEMATICS

Core Courses - ME010105 - GRAPH THEORY

2019 Admission Onwards

DA350B30

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define (a) graph (b) isomorphism of graphs (c) simple graph (d) induced subgraph of a graph
2. Show that the edges incident at a vertex of a graph G give rise to a complete subgraph of the line graph L(G).
3. State Whitney's characterization theorem of 2-connected graph and using this prove that a graph G with atleast three vertices is 2-connected if and only if any two vertices of G lie on a common cycle.
4. Define (a) cyclical edge connectivity of a graph
   (b) nonseparable graph
   (c) block of a graph
   (d) End block of a graph
5. Write Kruskal's algorithm for determining a minimum weight spanning tree in a connected weighted graph.
6. Explain Knight's Tour in a Chessboard.
7. Define chromatic number and chromatic partition of vertices of a graph G.
8. State Brook's theorem.
9. Define a planar graph.
10. What is the spectrum of K_n

(8×1=8 weightage)

Part B (Short Essay/Problems)

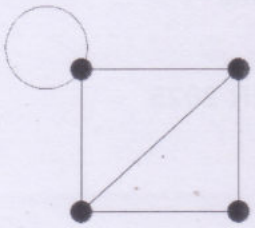
*Answer any **six** questions.*

*Weight **2** each.*

11. Define graphical sequence and write the necessary condition for $d=\{d_1, d_2, \ldots, d_n\}$ to be graphical. Show that $d=(\{7,6,3,3,2,1,1,1\})$ is not graphical.
12. Find the order and size of $G_1 \times G_2$

13. Show that if a simple cubic connected graph has a cut vertex then it has a cut edge.

14. Find the number of spanning trees of the following graph



15. Explain Konigsberg Bridge Problem.

16. For any simple graph G, prove that $2\sqrt{n} \leq \chi + \chi^c \leq n+1$ and $n \leq \chi\chi^c \leq ((n+1)/2)^2$

17. If G is simple connected planar bipartite graph with $n \geq 3$, then show that $m \leq 2n-4$

18. Show that $K_{3,3}$ is a nonplanar graph using the Jordan Curve Theorem.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. 1.Show that every vertex of a diconnected tournament T on n vertices with n≥3 is contained in a directed k-cycle, 3≤k≤n.
2.Show that for every positive integer n≥3 there exist a tournament not containing a spanning directed cycle.
3.Define (i) Directed graph
(ii) outneighbour of a vertex and inneighbourm of a vertex
(iii) dicomponent
(iv) tournament

20. 1. Show that every tree has a centre consisting of either a single vertex or two adjacent vertices.
2.Prove that a simple graph G is a tree if and only if any two distinct vertices are connected by a unique path.
3.Show that a connected graph G is a tree if and only every edge of G is a cut edge.

21. (a) Define closure of a graph. Prove that a graph has one and only one closure.
(b) Prove that if cl(G) is hamiltonian then G is hamiltonian.
(c) Give an example of a graph G such that cl(G)=G

22. State and prove Heawood's five color theorem.

(2×5=10 weightage)